

Comparing lattice Dirac operators with Random Matrix Theory*

F. Farchioni, I. Hip[†] and C. B. Lang

Institut für Theoretische Physik, Universität Graz, A-8010 Graz, Austria

We study the eigenvalue spectrum of different lattice Dirac operators (staggered, fixed point, overlap) and discuss their dependence on the topological sectors. Although the model is 2D (the Schwinger model with massless fermions) our observations indicate possible problems in 4D applications. In particular misidentification of the smallest eigenvalues due to non-identification of the topological sector may hinder successful comparison with Random Matrix Theory (RMT).

1. INTRODUCTION

In recent work we have been studying various aspects of the lattice Schwinger model [1,2]. This model is a 2D U(1) gauge theory of photons and one or more fermion species. Of particular interest is the situation of massless fermions. In the quantized theory chiral symmetry is broken by the anomaly. The one flavor-model should exhibit a bosonic massive mode.

For the non-perturbative lattice formulation chirality is a central issue. The Wilson Dirac operator explicitly breaks chiral symmetry. The Ginsparg-Wilson condition [3] defines a class of lattice actions with minimal violation of chirality. An explicit realization is Neuberger's overlap Dirac operator [4]. In another approach one attempts to construct so-called quantum perfect actions, or fixed point actions (classically perfect actions) [5], also obeying the Ginsparg-Wilson condition [6].

In the Schwinger model framework we have been studying several of these suggestions. In [7] the (approximate) fixed point Dirac operator was explicitly constructed. It has a large number of terms but has been shown to have excellent scaling properties for the boson bound state propagators. This is not the case for the Neuberger operator [2]; there scaling is not noticeably improved over the Wilson operator. The overlap operator has eigenvalues distributed ex-

actly on a unit circle in the complex plane; for the (approximate) fixed point operator our study shows small (with smaller $\beta = 1/g^2$ increasing) deviations from exact circularity. In both cases we could identify chiral zero modes. Their occurrence was strongly correlated to the geometric topological charge of the gauge configuration $\nu_{\text{geo}} = \frac{1}{2\pi} \sum_x \text{Im} \ln U_{12}(x)$ (henceforth called ν for brevity) with a rapidly improving agreement with the Atiyah-Singer Index Theorem (interpreted on the lattice) towards the continuum limit.

Studying the spectra of the Dirac operators suggests comparison with Random Matrix Theory (RMT)[8]. There the spectrum is separated in a fluctuation part and a smooth background. Exact zero modes are disregarded. The fluctuation part, determined in terms of the so-called unfolded variable (with average spectral spacing normalized to 1), is conjectured to follow predictions lying in one of three universality classes. For chiral Dirac operators these are denoted by chUE, chOE and chSE (chiral unitary, orthogonal or symplectic ensemble, respectively) [9]. Various observables have been studied in this theoretical context. Comparison of actual data should verify the conjecture and allows one to separate the universal features from non-universal ones. In particular it should be possible to determine in this way the chiral condensate.

On one hand the limiting value of the density for small eigenvalues and large volume,

$$-\pi \lim_{\lambda \rightarrow 0} \lim_{V \rightarrow \infty} \rho(\lambda) = \langle \bar{\psi} \psi \rangle, \quad (1)$$

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[†]Poster presented by I. Hip

provides such an estimate due to the Banks-Casher relation. This information is contained in the smooth average (background) of the spectral distribution. However, also the fluctuating part, in particular the distribution for the smallest eigenvalue $P(\lambda_{\min})$ contains this observable: Its scaling properties with V are given by unique functions of a scaling variable $z \equiv \lambda V \Sigma$, depending on the corresponding universality class. Usually this is the most reliable approach to determine Σ , which then serves as an estimate for the infinite volume value of the condensate in the chiral limit. This method does not involve unfolding, averaging or extrapolation.

Here we concentrate on our results for the staggered Dirac operator. It is anti-hermitian and (for $m = 0$) its spectrum is located on the imaginary axis, but it has no exact zero modes. RMT predictions for the staggered action and the trivial topological sector have been confirmed also in 4D lattice studies [10]. Here we emphasize, however, the rôle of non-zero topological charge.

2. METHOD AND RESULTS

In our study we construct sequences of (5000-10000) uncorrelated quenched gauge configurations for several lattices sizes (16^2 , 24^2 , 32^2) and values of β (2, 4, 6). For these sets we then determine the various Dirac operators and study their spectral distribution. This way we can compare directly the effect of identical sets of gauge configuration on the fermionic action. In [1] we discuss our results for the Neuberger- and the fixed point operator. Since these spectra lie on or close to a circle in the complex plane, one has to project them to the (tangential) imaginary axis. We find that they exhibit the universal properties of the (expected) chUE-class, unless the physical lattice volume is too small.

In Fig. 1 we demonstrate the relevance of topological modes. The e.v. distribution density is first shown without distinguishing between different ν and we notice a pronounced peak at small eigenvalues. Splitting the contributions according to $|\nu| = 0$ and 1 we observe, that the peak is due to the non-trivial sectors $\nu \neq 0$. The trivial sector has a behavior typical for the shapes pre-

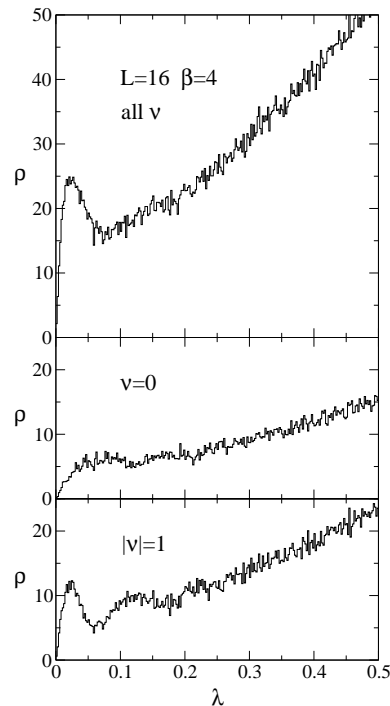


Figure 1. Distribution density of e.v.s of the staggered Dirac operator. (out of all 10000 configurations 29% have $\nu = 0$ and 45% have $|\nu| = 1$.)

dicted from chRMT. For larger β and V the peak becomes more pronounced, justifying the hypothesis that it represents the “would-be” zero modes.

Since RMT discusses the distribution *excluding exact zero-modes* we expect problems whenever one is in a situation without possibility to separate topological sectors (upper-most figure in Fig. 1), if one then tries to represent the distribution for the smallest observed eigenvalue by chRMT functions. This is demonstrated in Fig. 2 where we plot the histograms for the smallest and the 2nd smallest (shaded histogram) eigenvalues *in the $|\nu| = 1$ sector*. For small β , strong coupling, the histogram for the smallest e.v. behaves like the $\nu = 0$ sector prediction. For large β the 2nd smallest e.v. follows a distribution expected for the *smallest* e.v. in the $|\nu| = 1$ sector.

The level spacing distribution (determined in the unfolded variable) clearly has chUE (Wigner

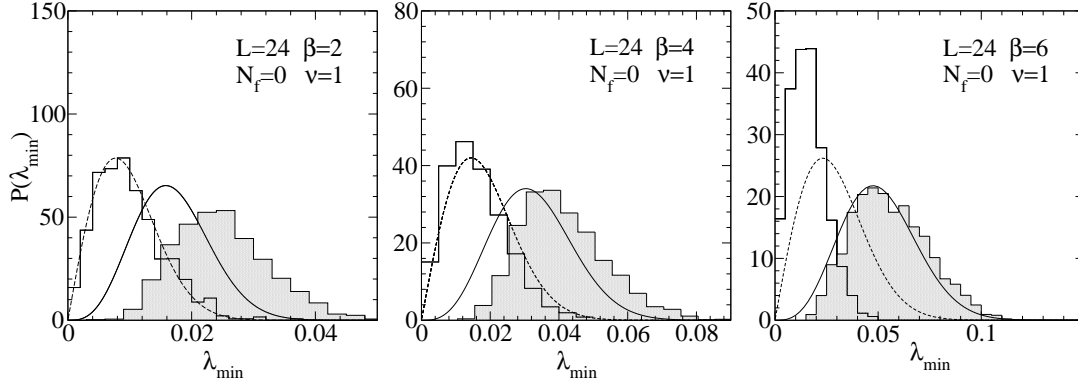


Figure 2. Distribution for the smallest and the 2nd smallest (shaded histogram) eigenvalues *in the* $\nu = 1$ sector, quenched ($N_f = 0$). The curves give the chUE predictions for the smallest e.v. in the $\nu = 0$ sector (dashed line) in the $\nu = 1$ sector (full line).

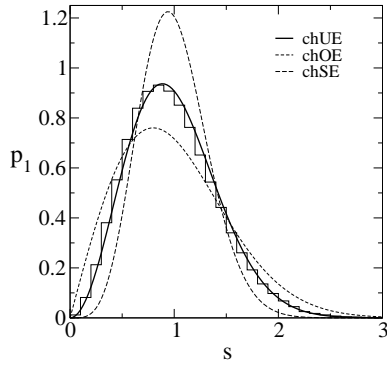


Figure 3. Level spacing distribution (for the unfolded variable s) for $L = 16$, $\beta = 4$, compared to chRMT predictions for chUE (full), chOE (dashed) and chSE (dotted).

surmise) shape (Fig. 3) for all sizes and β .

Having all eigenvalues we can of course calculate the fermion determinant for every gauge configuration and include dynamic fermions by explicit multiplication. These “unquenched” results will be presented elsewhere.

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